Springer

KSME

Journal of Mechanical Science and Technology 24 (10) (2010) 1957~1961
www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-010-0704-x

Dynamic response of a finite length euler-bernoulli beam on linear and nonlinear viscoelastic foundations to a concentrated moving force

Alkim Deniz Senalp[1], Aytac Arikoglu[1], Ibrahim Ozkol[1,*] and Vedat Ziya Dogan[1]

[1] *Department of Aeronautical Engineering, Istanbul Technical University, Istanbul, 34469, Turkey*

(Manuscript Received February 23, 2009; Revised March 18, 2010; Accepted June 28, 2010)

Abstract

In this paper the dynamic response of a simply-supported, finite length Euler-Bernoulli beam with uniform cross-section resting on a linear and nonlinear viscoelastic foundation acted upon by a moving concentrated force is studied. The Galerkin method is utilized in order to solve the governing equations of motion. Results are compared with the finite element solution for the linear foundation model in order to validate the accuracy of the solution technique. A good agreement between the two solution techniques is observed. The effect of the nonlinearity of foundation stiffness on beam displacement is analyzed for different damping ratios and different speeds of the moving load. The results for the time response of the midpoint of the beam are presented graphically.

Keywords: Euler-Bernoulli beam; FEM; Galerkin method; Moving force; Vibration; Viscoelastic foundation

1. Introduction

Recently, the investigation of the dynamic response of beams on viscoelastic foundations subjected to moving loads has been of great significance in railway engineering. Zheng et. al [1] gave a general solution dynamical problem of an infinite beam on an elastic foundation. Lee [2] investigated the dynamic response of a Timoshenko beam on a Winkler foundation subjected to a moving mass. Thambiratnam and Zhuge [3] studied the dynamics of beams on an elastic foundation subjected to moving loads by using the finite element method. They investigated the effect of the foundation stiffness, traveling speed and length of the beam on the dynamic magnification factor, which is defined as the ratio of the maximum displacement in the time history of the mid-point to the static mid-point displacement. Kim [4] investigated the vibration and stability of an infinite Euler-Bernoulli beam resting on a Winkler foundation when the system is subjected to a static axial force and a moving load with either constant or harmonic amplitude variations. The effects of load speed, load frequency, damping on the deflected shape, maximum displacement and critical values of the velocity, frequency and axial force are also studied. Kargarnovin and Younesian [5] studied the response of a Timoshenko beam with uniform cross-section and infinite length supported by a generalized Pasternak viscoelastic foundation subjected to an arbitrarily distributed harmonic moving load. Kargarnovin and Younesian [6] also analyzed the dynamic response of infinite Timoshenko and Euler-Bernoulli beams on nonlinear viscoelastic foundations to harmonic moving loads.

In this study, the dynamic response of a simply-supported, finite length, uniform cross-section Euler-Bernoulli beam resting on a linear and nonlinear viscoelastic foundation acted upon by a moving concentrated force is studied. In existing literature, research based on the response of beams on foundations assumes that the beam is infinite. In this study, an infinite track is replaced by a finite track. Since the beam is placed over a very stiff foundation, the moving load has a local effect and it is sufficient to analyze a small portion of the beam. The Galerkin method is used to solve the initial boundary value problem that governs the transverse vibration of the beam. Time response histories of the beam are graphically presented for various speeds of force. The effect of nonlinearity of the foundation stiffness is also investigated.

2. Theory and formulation

In Figs. 1 and 2, simply-supported, homogeneous, isotropic and constant cross-section beams of length L over viscoelastic foundations are shown. Linear and nonlinear foundation models are used. Viscoelastic foundations consist of dashpots and springs. In the literature, the railway track is usually assumed to be linear in order to simplify the track model, although the rail pad and ballast are actually non-linear. The beam is ini-

† This paper was recommended for publication in revised form by Associate Editor Eung-Soo Shin

* Corresponding author. Tel.: +90 2122853111, Fax: +90 2122852926
E-mail address: ozkol@itu.edu.tr

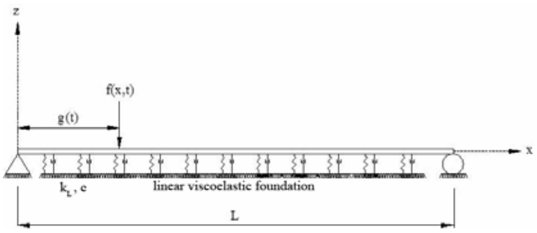

Fig. 1. Simply-supported beam on linear viscoelastic foundation.

tially assumed to be at rest and undeformed. The force f(x,t) is expressed as follows:

$$f(x,t) = \delta(x - g(t))P_0 \tag{1}$$

where δ is the Dirac-Delta function, P_0 refers to the constant force and g(t) represents the kinematics of the moving force as follows:

$$g(t) = vt \tag{2}$$

where v is the constant speed of the force. The Dirac-Delta function, δ(x), is thought of as a unit concentrated force acting at point x=0. The Dirac-Delta function is defined as:

$$\int_a^b \delta(x-\xi) f(x)dx = f(\xi) \text{ , for } a < \xi < b \ . \tag{3}$$

2.1 Linear foundation model

To compare the effects of the linearity and nonlinearity of the foundation, a linear foundation model is considered first. A linear viscoelastic foundation model is shown in Fig. 1.

The problem is governed by the following differential equation:

$$\begin{aligned} &EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} + k_L w(x,t) \\ &+c\frac{\partial w(x,t)}{\partial t} = \delta(x - vt)P_0 \end{aligned} \tag{4}$$

where EI, ρ, A, c and w(x,t) are the flexural rigidity, the density, the cross-sectional area, the damping coefficient and the transverse deflection of the beam at point x and time t, respectively. k_L is the linear foundation stiffness and c is the viscous damping coefficient of the foundation. The simply-supported beam boundary and initial conditions are:

$$\begin{aligned} &w(0,t) = w(L,t) = 0 \\ &\frac{\partial^2 w(0,t)}{\partial x^2} = \frac{\partial^2 w(L,t)}{\partial x^2} = 0 \end{aligned} \tag{5}$$

$$w(x,0) = \frac{\partial w(x,0)}{\partial t} = 0 \ . \tag{6}$$

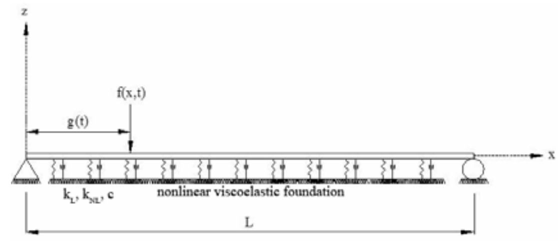

Fig. 2. Simply-supported beam on a nonlinear viscoelastic foundation.

2.2 Nonlinear foundation model

For this model, the viscoelastic foundation is modeled by the combination of linear and cubic nonlinear springs, where k_{NL} is the nonlinear part of the foundation stiffness and c is the damping coefficient. The problem is governed by the following differential equation:

$$\begin{aligned} &EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} + \\ &k_L w(x,t) + k_{NL} w^3(x,t) + c\frac{\partial w(x,t)}{\partial t} = \delta(x - vt)P_0 \end{aligned} \ . \tag{7}$$

The boundary and initial conditions are the same as given in Eqs. 5 and 6. Eq. 7 represents a nonlinear initial boundary value problem.

3. Solution method

3.1 Galerkin method

The Galerkin method is applied to Eqs. 4 and 7 to discretize the problem in a spatial coordinate and to obtain a system of ordinary differential equations in the time domain. The transverse displacement is assumed in the following form:

$$w(x,t) = \sum_{n=1}^{N} T_n(t) Sin(n\pi x / L) \ . \tag{8}$$

The basis functions are selected as *Sin(nπx/L)* in order to satisfy the boundary conditions in Eq. 5. By using Eq. 8 and Eq. 4, the Galerkin method can be applied for the linear foundation model as follows:

$$\begin{aligned} &\sum_{n=1}^{N}\int_0^L \left\{ \left[EI\left(\frac{n\pi}{L}\right)^4 + k_L \right] T_n(t) + \rho A T_n''(t) + cT_n'(t) \right\} \times \\ &Sin\left(\frac{n\pi x}{L}\right) Sin\left(\frac{m\pi x}{L}\right) dx = P_0 \int_0^L \delta(x - vt) Sin\left(\frac{m\pi x}{L}\right) dx \end{aligned} \tag{9}$$

which simplifies to the following form:

$$\begin{aligned} &\left[EI\left(\frac{m\pi}{L}\right)^4 + k_L \right] T_m(t) + \rho A T_m''(t) + cT_m'(t) = \\ &\frac{2P_0}{L} Sin\left(\frac{m\pi vt}{L}\right), \ for \ m = 1,2,3,...,N \end{aligned} \tag{10}$$

and the initial conditions in Eq. 6 become:

$$T_m(0) = T_m{}'(0) = 0 \;\; for \;\; m = 1,2,3,...,N \; . \tag{11}$$

The same procedure for the nonlinear foundation model is repeated, and the governing equations are derived.

$$\begin{aligned} &\left[EI\left(\frac{m\pi}{L}\right)^4 + k_L \right] T_m(t) + \rho A T_m{}''(t) + cT_m{}'(t) + \\ &\frac{k_{NL}}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} T_i(t)T_j(t)T_k(t)\big[3\delta(i+j-k-m) \\ &-\delta(i+j+k-m) - 3\delta(i+j-k+m)\big] \\ &= \frac{2P_0}{L} Sin\left(\frac{m\pi vt}{L}\right) \\ &for \;\; m = 1,2,3,...,N \end{aligned} \tag{12}$$

The initial conditions in Eq. 11 apply to both linear and nonlinear foundation models.

3.2 Finite element method

In order to validate the results obtained with the Galerkin method, the displacement field for the linear foundation model is evaluated with FEM. For the Euler-Bernoulli beam, the element mass and stiffness matrices are given as follows:

$$M_e = \frac{\rho A L_e}{420}\begin{bmatrix} 156 & 22L_e & 54 & -13L_e \\ 22L_e & 4L_e^2 & 13L_e & -3L_e^2 \\ 54 & 13L_e & 156 & -22L_e \\ -13L_e & -3L_e^2 & -22L_e & 4L_e^2 \end{bmatrix} \tag{13}$$

$$K_e = \frac{EI}{L_e^3}\begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & 4L_e^2 & -6L_e & 2L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & 2L_e^2 & -6L_e & 4L_e^2 \end{bmatrix} . \tag{14}$$

The element force vector for a point load is given by:

$$F_e = P_0\begin{Bmatrix} 1 + 2t^3v^3/L_e^3 - 3t^2v^2/L_e^2 \\ tv + t^3v^3/L_e^2 - 2t^2v^2/L_e \\ 3t^2v^2/L_e^2 - 2t^3v^3/L_e^3 \\ t^3v^3/L_e^2 - t^2v^2/L_e \end{Bmatrix} \tag{15}$$

where L_e is the length of an element. The element damping matrix and the element stiffness matrix for the foundation are derived from the mass matrix by replacing the coefficient ρA with c and k_L, respectively. The element matrices are joined to form the global matrices and then the boundary conditions are applied. The element force vector is taken, as in Eq. 15, if the load is on the element, otherwise it is taken as zero.

Table 1. Properties of UIC60 rail.

Young's modulus (N/m^2)	21×10^{10}
Area moment of inertia (m^4)	3.055×10^{-5}
Cross sectional area (m^2)	7.69×10^{-3}
Density of the material (kg/m^3)	7850
Length of the beam (m)	50
Linear spring stiffness per length (Linear foundation) (N/m^2)	1.386×10^8
Linear spring stiffness per length (Nonlinear foundation) (N/m^2)	3.503×10^7
Nonlinear spring stiffness per length (Nonlinear foundation) (N/m^4)	4.01×10^{14}
Moving load (N)	65×10^3

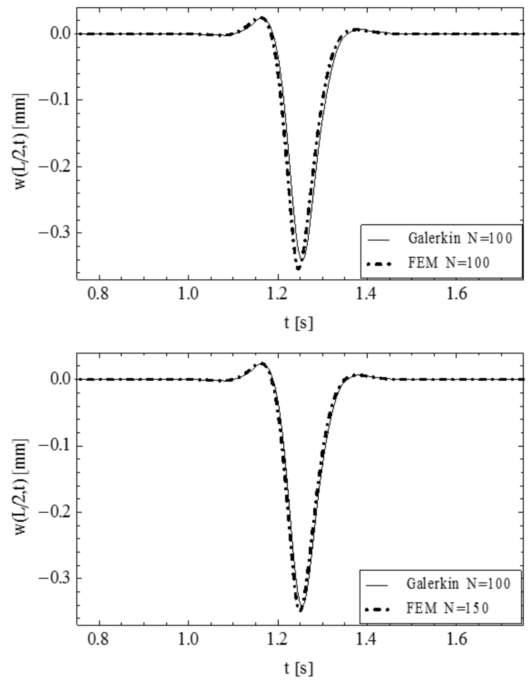

Fig. 3. Time response of beam evaluated with Galerkin method and FEM (v=20 m/s, ξ=9.47).

4. Results and discussion

A finite beam of 50 meters in length is considered long enough to replace an infinite beam. The material properties for the rail, foundation and load are presented in Table 1 [6].

For each velocity case, damped and over-damped dynamic responses of linear and nonlinear viscoelastic foundation models are investigated. For the linear foundation model, damping ratios are determined by considering the critical damping coefficient (c_{cr}). The damping ratio (ξ) is expressed as follows:

$$\xi = c / c_{cr} \tag{16}$$

$$c_{cr} = 2\sqrt{k_L \rho A} \tag{17}$$

Fig. 3 shows the comparison between the results of the

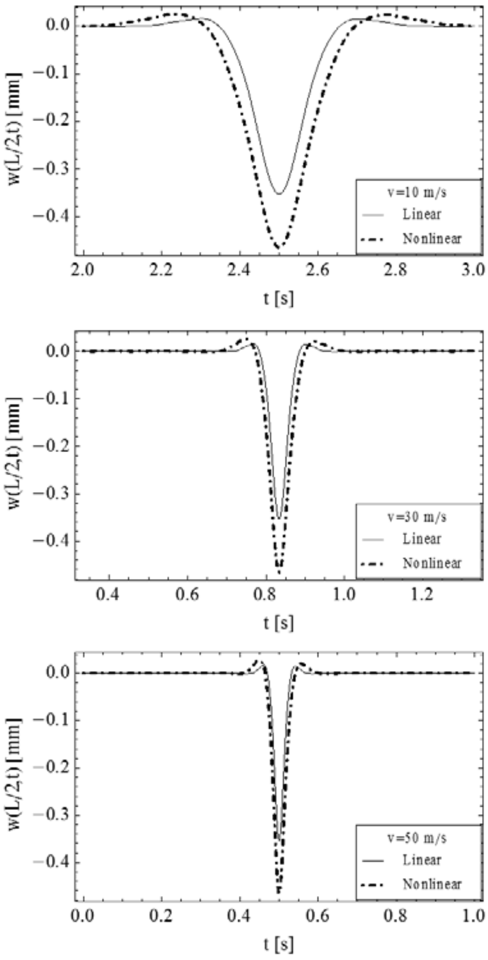

Fig. 4. Time response diagrams of beams with two distinct foundation models for the damping ratio ξ=0.5.

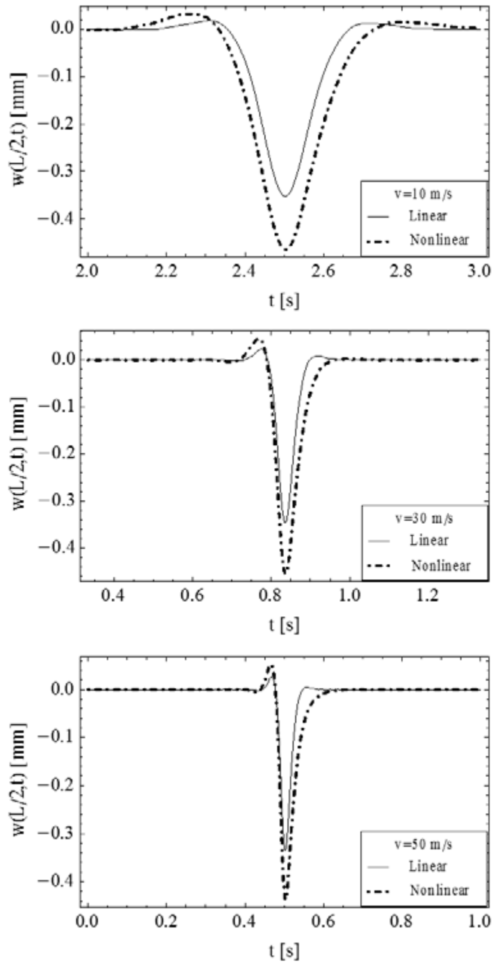

Fig. 5. Time response diagrams of beams with two distinct foundation models for the damping ratio ξ=5.

Galerkin and finite element methods for a linear foundation. Since the beam model is simply supported at both ends, and because of the symmetry, maximum deflections will occur at the mid-span of the beam, L/2. Therefore, the results are presented for this point.

The results show that a perfect agreement between these two methods is reached as the number of elements considered in the finite element calculation increases.

Figs. 4-6 show a one second time portion of the mid-point vertical deflections with time. The velocity is taken between 10 to 50 m/s.

Fig. 4 shows the effect of moving load speed on the vertical deflection for a relatively small damping ratio. The local effect of the moving load is clearly seen from the figure, especially at higher speeds. Vertical mid-point deflections are virtually negligible until the moving loads approach the midpoint.

Another important point is that the transverse vibration amplitude decreases with increasing speed. The nonlinear and equivalent linear models are in good agreement for small deflections. However, the effect of nonlinearity starts to dominate for larger deflections. It can also be seen from the figures that for ξ=0.5, the transverse displacement of the beam is close to being symmetrical. Moving load speed and nonlinearity have similar effects for higher damping ratios, as shown in Figs. 5 and 6.

However, as the damping ratio increases, the symmetry of the displacement is distorted. The magnitude of the vibration amplitude decreases with increased damping due to the loss of kinetic energy in the form of heat energy. Kargarnowin et al. [6] studied the effect of two successive moving loads by using an FEM model. The results presented in Figs. 4-6 show very good agreement with the results in [6] in terms of both the magnitude and the distribution of deflection.

5. Conclusions

In this study, the dynamic response of a simply-supported, finite length Euler-Bernoulli beam with uniform cross-section resting on a linear and nonlinear viscoelastic foundation acted upon by a moving concentrated force is studied. An infinite track with nonlinear foundation is replaced with a finite one. This boundary value problem was solved for linear and

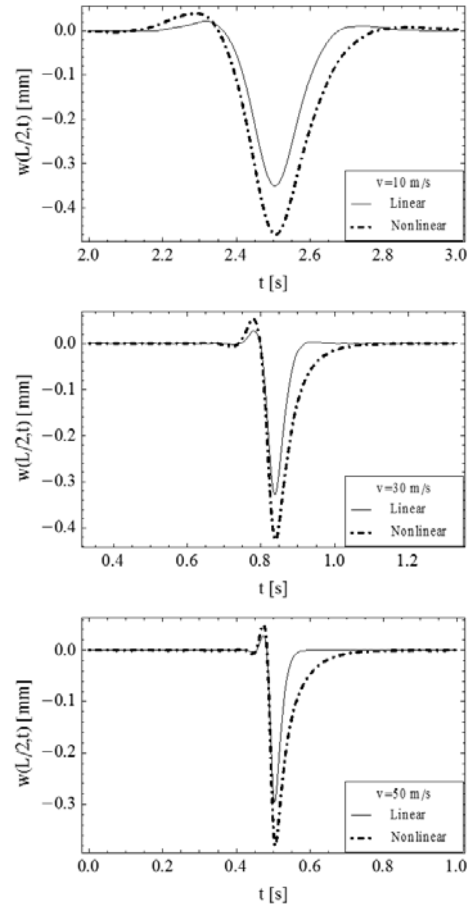

Fig. 6. Time response diagrams of beams with two distinct foundation models for the damping ratio ξ=10.

nonlinear cases by applying the Galerkin method. The time responses of the beams with linear and nonlinear cases are presented for various speeds of moving force. The effects of nonlinearity in stiffness can easily be observed from the figures. From Figs. 4-6, one can deduce the following results:

- As the force speed increases, the dynamic response of the beam decreases for both linear foundation models.
- As the damping ratio (ξ) increases, the dynamic deflections decrease for both linear and nonlinear cases.
- For the nonlinear foundation model, the dynamic response of the beam is always greater when compared to the linear foundation model.
- The distribution of deflection is symmetrical for small values of the damping ratio and this symmetry is distorted with increasing damping.

References

[1] D. Y. Zheng, Y. K. Cheung, F. T. K. Au and Y. S. Cheng, Vibration of multi-span non-uniform beams under moving loads by using modified beam vibration functions, *Journal of Sound and Vibration,* 212 (3) (1998) 455-467.

[2] H. P. Lee, Dynamic response of a Timoshenko beam on a Winkler foundation subjected to a moving mass, *Applied Acoustics*, 55 (3) (1998) 203-215.

[3] D. Thambiratnam and Y. Zhuge, Dynamic analysis of beams on an elastic foundation subjected to moving loads, *Journal of Sound and Vibration,* 198 (1996) 149-169.

[4] Seong-Min Kim, Vibration and Stability of axial loaded beams on elastic foundation under moving harmonic loads, *Engineering Structures,* 26 (2004) 95-105.

[5] M. H. Kargarnovin and D. Younesian, Dynamics of Timoshenko beams on Pasternak foundation under moving load, *Mechanics Research Communications*, 31 (2004) 713-723.

[6] M. H. Kargarnovin and D. Younesian, Response of beams on nonlinear viscoelastic foundations to harmonic moving loads, *Computers & Structures,* 83 (2005) 1865-1877.

A. Deniz Senalp received his B.S. degree in Aeronautical Engineering from Istanbul Technical University, Turkey, in 2005. He then received his M.S. degree from the same department of ITU in 2008. A. Deniz Senalp is currently a Research Assistant and PhD. Student in ITU Aeronautics and Astronautics Department.

Aytac Arikoglu received his B.S and M.S. degrees from Istanbul Technical University in 2002 and 2004 respectively. He is currently a PhD candidate and a research assistant in the Faculty of Aeronautics and Astronautics in ITU. His research areas are applied mathematics, thermodynamics and vibration analysis of sandwich structures.

Ibrahim Ozkol is working in Istanbul Technical University at the faculty of aeronautics and astronautics as a Professor. His research interests are fluid mechanics, dynamics, kinematics and control of parallel mechanisms.

Vedat Z. Dogan received the BS degree from Istanbul Technical University, Turkey; the M.S. and Ph.D. degrees from Columbia University in New York, USA. He currently works as an Associate Professor in the Department of Aeronautical Engineering at Istanbul Technical University, Istanbul, Turkey. His current research interests include structural dynamics, plates and shells, and random vibrations.